A participation framework to inform mathematics courses for prospective elementary teachers

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Outline

- A brief overview of my research agenda
- My experience teaching content courses for prospective elementary teachers
- A participation framework
- Implications for teacher education

My research

I have done research in different areas:

- Preservice elementary teachers' understanding of and beliefs about mathematics;
- Mexican American parents' perceptions about the teaching and learning of mathematics;
- Linking out-of-school experiences with in-school mathematics;
- Obstacles and affordances for the participation of non-dominant students in the mathematics classroom.

It all started when teaching content courses for preservice elementary teachers

- What I did….
  - Problem-solving type tasks; group work; encouraged different approaches and discussion.

- What I noticed….
  - A difference in how students “successful at math” approached problems and how students “less successful at math” approached them.

An example

- A watermelon and two cantaloupes cost \$4.65. Three watermelons and two cantaloupes cost \$10.15. How much does one watermelon cost?
- David, a preservice elementary teacher, solves it by reasoning that the difference between \$10.15 and \$4.65 corresponds to two watermelons, so one watermelon will cost \$2.75.
- However, he seems hesitant and checks his reasoning several times. Why?

- The watermelon looks too expensive to him. (This was in the mid 80s)
- David: *Actually I probably should use algebra because I'd come with proper numbers; this [his solution] could be wrong, I'm not so confident*.

- On the other hand, Barb, another preservice elementary teacher, solved the watermelon problem through algebra in a procedural way:
  - “I did it this way because the first one [equation] is $w + 2c = 4.65$, and I was taught that you always solve for the one that doesn’t have any number by it.”

- She also commented that she never analyzed for which unknown to solve if the problem asked for only one of them. She would solve it for both.

- Barb was uncomfortable using methods that “did not look like math.”
  - “When I think of math, I think of a formula that goes with it. A story problem, you set it up, you solve it, that’s math.”

- Needless to say that Barb was not concerned about expensive watermelons…

- Who would we'd rather have as preservice teachers and then teachers? David? Barb?

- Situations like these led me to the literature on situated cognition; street math/school math; the concept of context; as well as to the concept of valorization of knowledge: what knowledge do we value in our mathematics content courses? And whose knowledge do we value?

Another example

- Combination problems are quite typical when talking about multiplication in the content courses for preservice elementary teachers; for example, shirt and shorts; shirts, shorts, and shoes; pizza with types of crust, toppings, etc.

- In one of the first courses I taught I gave them the following problem

A restaurant serves salads with a choice of one of three vegetables (Romaine, Boston, Spinach), a choice of one of four dressings (Italian, French, Thousand Island, Blue Cheese), a choice of three kinds of bread (White, Whole wheat, Rye), and the option of having olives and/or anchovies in your salad.

If I wanted to try each of their different salads (one per day), how many times would I have to go to that restaurant?

I was expecting most likely a tree diagram (we had been working with those) and something like: 3 x 4 x 3 x 2 x 2 or 3 x 4 x 3 x 4

Vicky’s solution

What she did:

A tree diagram and 3 x 4 x 4 = 48 as her answer

What she wrote:

“The bread had nothing to do with the question or the vegetables.”

Some questions to consider

- What if Vicky had not been able to show/ explain her work? (e.g., multiple choice setting)

- Who seemed to have a stronger understanding of what it means to solve a problem, David or Barb?
- What do / should we do as instructors of content courses for elementary teachers? What messages do we send about what are "good" ways to do math?

- "There is hope yet when I can legally use my methods to solve a problem." [Vicky]

About participation

- Think about your students, your classroom… what may be some affordances and barriers for certain students to participate more or less?
- A question for me is, “does everybody have a voice in the mathematics classroom?”
- I am particularly concerned when working with learners (parents, children, college students) whose knowledge has historically not been recognized or valued by learning institutions.

A participation framework (Civil, 2014)

- Status: What does it mean to be good at math?
- Task: Whose knowledge and experiences are represented?
- Approach: Whose and what approaches are valued?
- Communication: Which language(s) and forms of communication are privileged in mathematical discussions?

- In what follows I illustrate two components of this framework by drawing on my work in Mexican American communities with teachers, parents and children.*

- I use this participation framework with a focus on an asset-based view of teaching and learning and to push us to think about what are possible implications for our content courses.

- *From (Civil & Hunter, 2021)

From the participation framework: approach

- Whose / what approaches get valued?

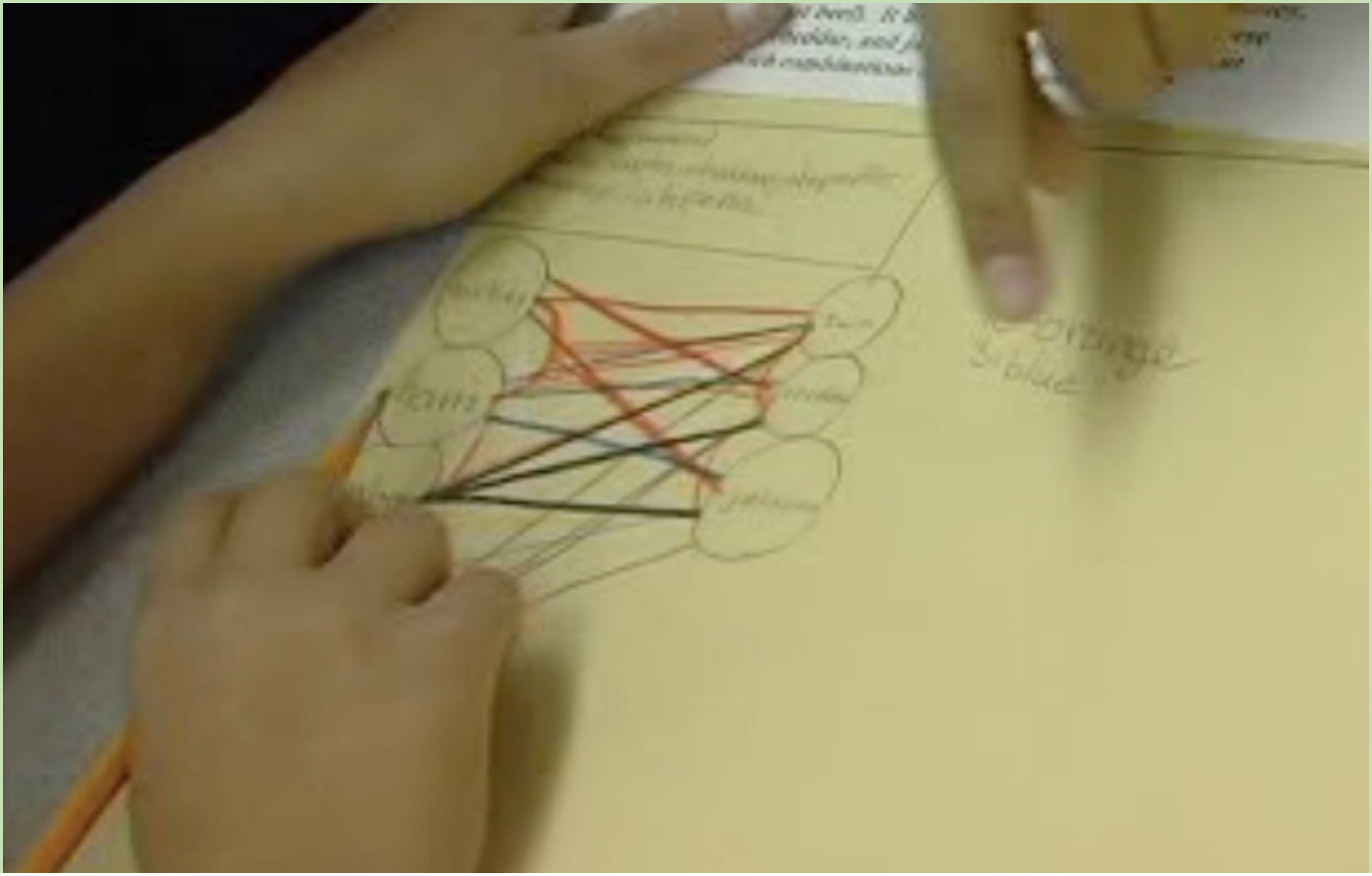
- Context: a $4^{th}/5^{th}$ grade classroom (ages 9-10); instruction in English but most children bilingual (English-Spanish).

- The problem:
- A restaurant serves different types of sandwiches. It has four different types of meat: turkey, ham, baloney, and roast beef. It has three different types of cheese: cheddar, jalapeño, and Swiss. How many different sandwich combinations can the restaurant sell?

Group work

- Students are working in groups of 2 or 3 on this problem; they come up with different representations. Most of the groups interpreted the problem as intended by the teacher, that is leading to 4 x 3 , so 12 different sandwich combinations.

- Ignacio and Pearl, have a different interpretation, allowing for many more combinations (e.g., two kinds of meat and one cheese, three kinds of meat and two kinds of cheese, etc.).



Sophisticated collaboration (Rogoff and colleagues; Alcalá, Rogoff & López Fraire, 2018)

- Ignacio and Pearl work together on this problem in a very collaborative way:
  - They are both hunched over the poster paper, taking turns with the colored pencils to indicate combinations.
  - They do not talk past each other: they talk “together” almost as if completing each other’s sentences at times.
  - When presenting to the whole class, they support each other.

The teacher's message: "it has to make sense to you"

- As the different groups were working on the problem, the teacher walked around mostly seeing what the students were doing.
- The teacher gave much freedom to students to approach the problems anyway they wanted. Hence, the many different representations we saw.
- The teacher believed in her students as mathematical thinkers.
- In particular, Pearl's and Ignacio's approach was not something the teacher expected and that became clear as she tried to understand their explanation. But she invited them to present to the whole class and gave them the space and time to present a very different approach.

- What did the teacher do so that students felt comfortable to approach problems any way they wanted?

- What do we need to do in our courses for elementary teachers so that when they have students like Pearl and Ignacio in their classes (which they will since they are in every class), they are open to their mathematical approaches and invite them to share their thinking with the class, even though it may be very different from what the teacher had in mind and may even be “unclear” to the teacher?

From the participation framework: Communication

- Which language(s) and forms of communication are privileged in mathematical discussions?
- The example we present here is intended to call for the need for teachers / schools to learn about and from children's mathematical interactions with their family members, in particular their parents / guardians.
- Context: mathematics workshop for families; sites for learning about (immigrant) parents' perceptions of mathematics, their uses of mathematics in everyday life, their interactions with their children about mathematics.

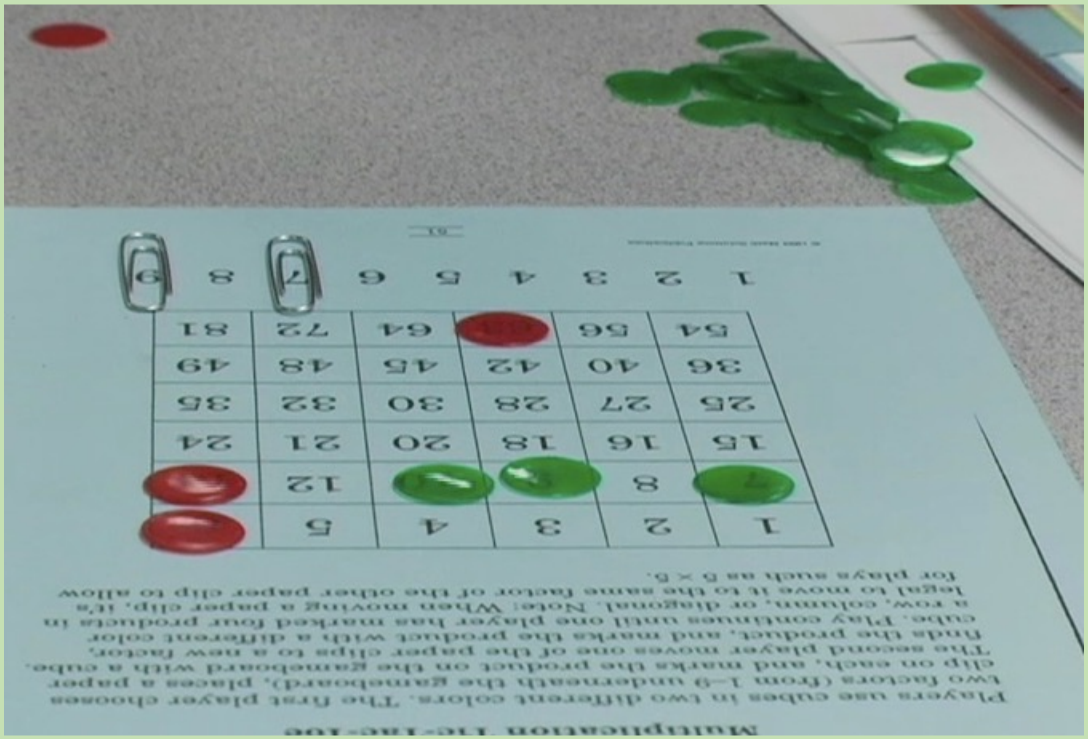
Multiplication Tic-Tac-Toe (4 in row, column, diagonal)

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1 2 3 4 5 6 7 8 9

- A father and his son (Carl, 10 years old) are playing the game. Carl knows how to play the game from class.
- Carl has started with clips on 2 and 5, so green counter on 10; then the father has moved the clip from 5 to 7, so his red counter is on 14; Carl then moves the clip from 2 to 1, and puts counter on 7

- The father does not seem quite sure what the goal of the game is and I help explaining it a little bit



After that exchange the father understands the goal of the game and moves clip to 6 (so 1 x 6); then Carl moves to 9 (so 1 x 9); he only needs one more for 4 in a row, (1 x 8). But his father moves the clip on 1 to 7, so 7 x 9

- The father supports his son’s learning by playing the game and in fact making it last longer by moving the paper clip so that the action moves to a different part of the board and also provides for more “interesting” multiplications.

Father: No, lo que voy a hacer es mover allá para que jueges por otra parte... Así juegas más [*No, what I’m going to do is to move over there so that you can play in another section... That way you play more*]

[Father moves the paper clip from 1 to 7 leading to 7 x 9]

Listening to and watching the interactions

- Father speaks in Spanish; Carl uses Spanish to talk to his father; translanguaging when doing math (e.g., finding 7 x 9).
- Throughout the game there is a lot of humor and friendly teasing.
  - No, eres mi papá, no me bloquees [*No, you're my dad, don't block me*]
- There is also Carl proudly showing his father how he figures out 7 x 9, sharing with his dad how they are learning in school; a more conceptually based method instead of memorization. Carl splits 9 into 4 plus 5 and then does 7 x 5 which he knows is 35 and for 7 x 4 he does 14 and 14; he adds 35 and 28 and finds 63.

Implications

- There is much to learn from how parents and children interact around mathematics; the example I just presented shows a lot of humor, encouragement, and pride in sharing how the child learned.
- There is also much to learn from how children from diverse cultural backgrounds interact; the work of Rogoff and colleagues on sophisticated collaboration among Mexican heritage children in the US is very interesting and certainly something that I have seen in my work in Mexican American communities.

- Throughout our work there is an emphasis on the concept of family; this is an important concept in the literature on working with Latinx students at all levels (college included).
- By creating a space in the classroom for children to bring their cultural ways of being, it allows for children’s sense of belonging at school.
- This, I argue is true for college students too, thus the preservice teachers in our courses→ what should we do in our courses to encourage students to bring their own cultural ways of being to the mathematics classroom?

Let’s look at the framework again

- Status: What does it mean to be good at math?
- → how do we decide this in our teaching?

- Task: Whose knowledge and experiences are represented?
- → how can we learn from our students’ experiences and activities, their funds of knowledge, to try to develop tasks that represent them? (e.g., what do you notice? What do you wonder?)

- Approach: Whose and what approaches are valued?
- → how committed are we to encourage different approaches to solving problems?

- Communication: Which language(s) and forms of communication (interaction) are privileged in mathematical discussions?
- → how can we build on the concept of collaboration, which some / many of our students naturally do in their communities?
- → how can we create a family feeling, in which their cultural resources (e.g., language, humor) support the mathematical interactions?

In closing, I suggest that we

- Analyze participation in our mathematics content courses with the goal to change participation structures if needed.
- Think about the tasks we pose, are they targeting/ representing particular areas of interest? Particular mathematical strengths at the expense of others?
- Examine the role that our views of what counts as “good” mathematical communication play in our interactions with students.

Thank you!

References

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